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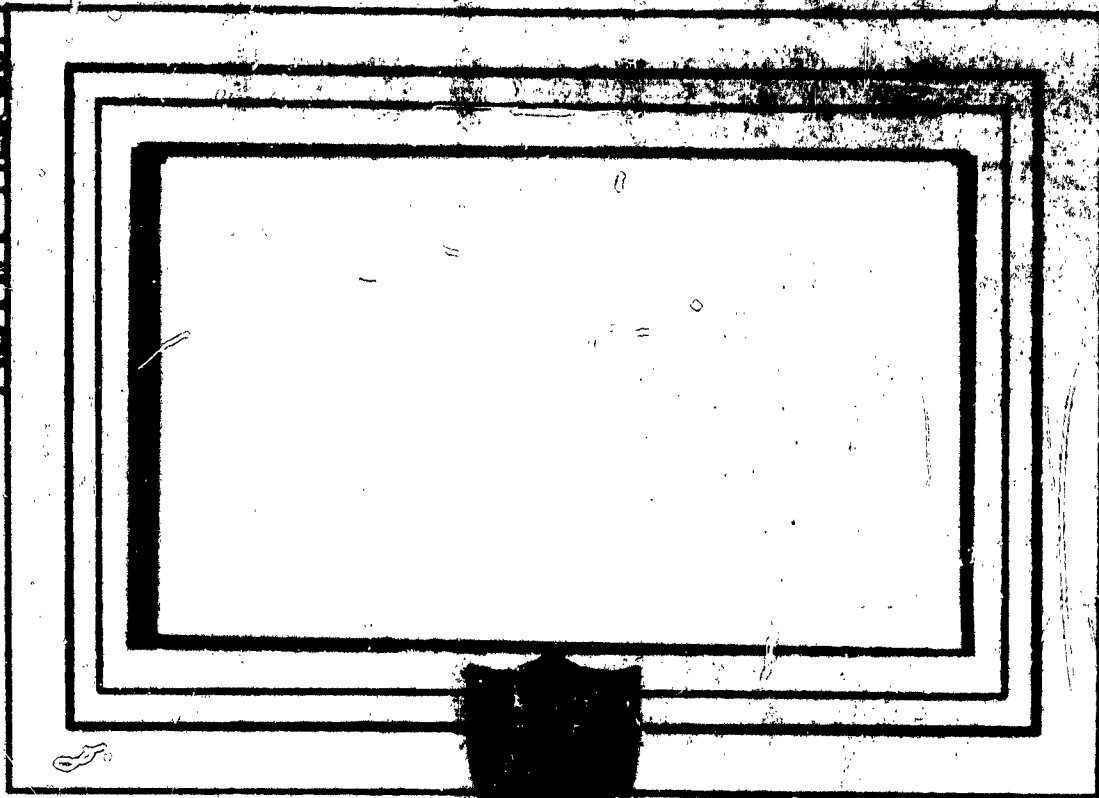FOR
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PRINCETON UNIVERSITY

DEPARTMENT OF AERONAUTICAL ENGINEERING

ON THE FLOW AT THE REAR OF A TWO-DIMENSIONAL
SUPERSONIC AIRFOIL WITH THICKNESS
by
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ON THE FLOW AT THE REAR OF A TWO-DIMENSIONAL
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A treatment of the problem of the flow deflection immed-
iately rearward of a two-dimensional supersonic airfoil was
given by Kahane and Lees[1] in 1948. Their analysis was carried
out on the basis of oblique shock and Prandtl-Meyer wave theory
utilizing the fact that the dynamic and static pressure change
through isentropic and shock waves can be expressed as a power
series in the flow deflection angle. Since to the writer"s
knowledge this is the only place where the dynamic pressure
coefficients have been published, it was thought worthwhile
to point out that one of the coefficients in the expansion
for an isentropic wave (also appearing in the shock expansion)
appears to be incorrect.

The authors expressed the change in dynamic pressure
through an isentropic wave as

$$\Delta q = q_1 \left(\pm G_1 \theta + G_2 \theta^2 \pm G_3 \theta^3 + \ldots \right)$$

and through a shock wave as

$$\Delta q = q_1 \left(\pm G_1 \theta + G_2 \theta^2 \pm (G_3 - H) \theta^3 + \ldots \right)$$

where $q = \rho w^2 / 2$ (ρ = density, w = flow velocity), θ is the
angle of deflection, positive in a counterclockwise sense, and
q_1 is the dynamic pressure ahead of the wave. The corrected
value for G_3 should be

$$G_3 = \frac{\frac{(3-\gamma)(2-\gamma)}{12} M^{10} + \frac{(-3\gamma^2 + 17\gamma - 36)}{12} M^8 + \frac{(21-4\gamma)}{3} M^6 - 8M^4 + \frac{14}{3} M^2 - \frac{4}{3}}{(M^2-1)^{7/2}}$$

where M is the flow Mach number ahead of the wave.

Determination of the flow angle immediately rearward of a flat-plate airfoil at angle of attack, and for a thick airfoil at angle of attack where it was assumed that the configuration was such that four oblique shock waves were attached to the nose and tail of the airfoil, (see Fig. 1), was carried out. Fortunately the calculations did not involve the use of the incorrect coefficient G_3 directly. In spite of this however, for the thick airfoil shown in Fig. 1, the analytic
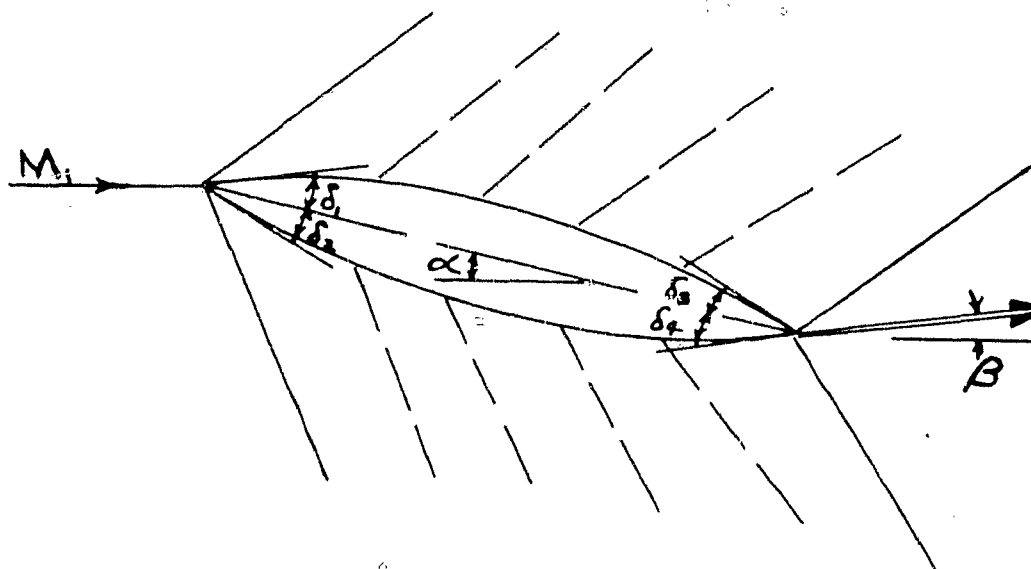

Fig. 1. The thick airfoil.

formulation of the flow deflection angle β to first order appears to be incorrect.

Recalculation of β gives

$$\beta = \frac{D}{2C_1}\left\{(\delta_3+\alpha)^3 + (\delta_1-\alpha)^3 - (\delta_4-\alpha)^3 - (\delta_2+\alpha)^3\right\}$$

$$+ \frac{E}{2C_1}\left\{(\delta_3+\alpha)^4 + (\delta_1-\alpha)^4 - (\delta_4-\alpha)^4 - (\delta_2+\alpha)^4\right\}$$

$$+ \frac{H}{2}\left\{(\delta_2+\alpha)^4 - (\delta_1-\alpha)^4\right\} - \frac{\left(\frac{dD}{d\theta}+DG_1\right)\left\{(\delta_3+\alpha)^4 - (\delta_4-\alpha)^4\right\}}{2C_1}$$

where D, C_1 and E are the coefficients of θ in the following expansion for the static pressure change through an oblique shock wave:

$$\Delta p = \rho_1 \left(\pm C_1 \theta + C_2 \theta^2 \pm (C_3 - D) \theta^3 + (C_4 - E) \theta^4 \pm \cdots \right)$$

The required coefficients for the determination of β are functions only of the free stream Mach number M_1, and the ratio of specific heats γ. Their values (other than E) are given in Ref. 1, while (C_4-E) may be found in Ref. 2. The computation of C_4 and hence E is easily carried out by use of the results of Miles[3].

REFERENCES

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2. Hilton, W.F.: "High-Speed Aerodynamics", Longmans, Green and Co., New York, (1951), p. 226.

3. Miles, E.R.C.: "Supersonic Aerodynamics", McGraw-Hill Co., Inc., New York, (1950), pp. 245-246.